Sebenta
Estatística 2

COMISSÃO DE 2º ANO 2016/2017

Este é um trabalho realizado por alunos, pelo que não está livre de conter gralhas ou falta de informação; torna-se, assim, essencial fazer uma análise crítica à sua leitura, tendo em conta a matéria lecionada nas aulas. Qualquer correção deverá ser enviada para comissao2ano@aefep.pt

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a) Conticidade

$$E(\hat{\mu}_1) = E(\overline{X}_{3000}) = E\left(\frac{1}{3000}\sum_{i=1}^{3000} x_i\right)$$

$$= \frac{1}{3000} E\left(\sum_{i=1}^{3000} x_i\right)$$

i.i.d

$$= \frac{1}{3000}\sum_{i=1}^{3000} E(X) = \frac{1}{3000}\sum \mu = \frac{1}{3000} \cdot 3000 \cdot \mu$$

$$= \mu \rightarrow \hat{\mu}_1 \text{ é côntico}$$

$$E(\hat{\mu}_2) = E(\overline{X}_{1000}) = \mu$$

Seja qual for a dimensão da amostra é sempre um estimador contiado p/ a média da população.

Consistência

$$\lim_{n \to +\infty} E(\overline{X}_n) = \mu \quad e \quad \lim_{n \to +\infty} V(X) = \mu$$

$$\lim_{n \to +\infty} E(\overline{X}_n) = \lim_{n \to +\infty} \mu = \mu \cdot 1 = \mu$$

$$\lim_{n \to +\infty} V(X) = \lim_{n \to +\infty} \frac{\sigma^2}{n} = 0$$ → pq são números

$$V(\hat{\mu}_1) = V(\overline{X}_{3000}) = V\left(\frac{1}{3000}\sum x_i\right)$$

$$= \frac{1}{3000^2} V\left(\sum x_i\right) = \frac{1}{3000^2}\sum V(x_i) = \frac{1}{3000^2}\sum \sigma^2$$

$x_1, \dots x_n$ independentes — i.i.d

$$= \frac{1}{3000}\sigma^2 = \frac{\sigma^2}{n}$$

As 2 condições suficientes p/ a consistência estão satisfeitas

eficiência relativa $\longrightarrow = \dfrac{EQM(\hat{\mu}_1)}{EQM(\hat{\mu}_2)} = \dfrac{Var(\hat{\mu}_1) + Erro^2(\hat{\mu}_1)}{Var(\hat{\mu}_2) + Erro^2(\hat{\mu}_2)}$

$eR = \dfrac{V(\hat{\theta}_1)}{V(\hat{\theta}_2)} = \dfrac{\frac{\theta^2}{3000}}{\frac{\theta^2}{1000}} = \dfrac{1}{3} < 1$

0 pq são cênticos

Logo $\hat{\mu}_1$ é + eficiente

b) $E(\hat{\mu}^*) = E\left(\frac{1}{2}(\hat{\mu}_1 + \hat{\mu}_2)\right)$

$= \frac{1}{2} E(\hat{\mu}_1) + \frac{1}{2} E(\hat{\mu}_2)$

$= \frac{1}{2}\mu + \frac{1}{2}\mu = \mu$ // (cêntico)

$V(\hat{\mu}^*) = V\left(\frac{1}{2}(\hat{\mu}_1 + \hat{\mu}_2)\right)$

$= \frac{1}{4} V(\hat{\mu}_1 + \hat{\mu}_2)$

$= \frac{1}{4}\left[V(\hat{\mu}_1) + V(\mu_2) + \underbrace{2\,cov(\hat{\mu}_1, \hat{\mu}_2)}_{0 \text{ pq são indep}}\right]$

$= \frac{1}{4}\left(\frac{\theta^2}{3000} + \frac{\theta^2}{1000}\right)$

$= \frac{1}{3000}\theta^2$

$eR = \dfrac{V(\hat{\mu}^*)}{V(\hat{\mu}_1)} = \dfrac{\frac{1}{3000}\theta^2}{\frac{1}{3000}\theta^2} = 1 \rightarrow$ eficiência igual

Não há ganho nenhum em proceder à combinação linear

2

$\hat{\theta} = c_1 \hat{\theta}_1 + c_2 \hat{\theta}_2$

$E(\hat{\theta}) = E(c_1 \hat{\theta}_1 + c_2 \hat{\theta}_2)$

$= E(c_1 \hat{\theta}_1) + E(c_2 \hat{\theta}_2)$

$= c_1 E(\hat{\theta}_1) + c_2 E(\hat{\theta}_2)$

$\downarrow$

i. id.

$= c_1 E(\hat{\theta}) + c_2 E(\hat{\theta})$

$= (c_1 + c_2)\theta$

So $c_1 + c_2 = 1$

$= \theta \Rightarrow$ cêntico//

③

$p \rightarrow$ proporção de pessoas que possuem automóveis marca A nessa cidade

$$x_i \rightarrow \begin{cases} 1, & \text{se a i-ésima pessoa possui autom. marca A} \\ 0, & \text{caso contrário.} \end{cases}$$

$\boxed{n = 100}$

$$\hat{p}_1 = \frac{1}{100} \sum_{i=1}^{100} x_i \qquad \hat{p}_2 = \frac{x_1 + 4x_{100}}{5}$$

a) Centricidade

$$E(\hat{p}_1) = E\left(\frac{1}{100} \sum_{i=1}^{100} x_i\right) = \frac{1}{100} \sum E(x_i)$$

$$\underset{\substack{\downarrow \\ \text{i.i.d} \\ \text{logo } x_i = X}}{=} \frac{1}{100} \sum_{i=1}^{100} E(X) = \frac{1}{\cancel{100}} \cdot \cancel{100} \cdot p = p$$

$$E(\hat{p}_2) = E\left(\frac{x_1 + 4x_{100}}{5}\right) = \frac{1}{5} E(x_1) + \frac{4}{5} E(x_{100})$$

$$\underset{\substack{\downarrow \\ \text{i.d.}}}{=} \frac{5}{5} E(X) = \frac{5}{5} p = p$$

Como $\hat{p}_1$ e $\hat{p}_2$ são cêntricos, a eficiência pode ser medida pela variância.

$$V(\hat{p}_1) = V\left(\frac{1}{100} \sum_{i=1}^{100} x_i\right) = \frac{1}{10000} V(\sum x_i)$$

$$= \frac{1}{10000} \sum V(x_i) \underset{\substack{\downarrow \\ \text{i.d.} \\ V(x_i) = p(1-p)}}{=} \frac{1}{1000} \sum p(1-p) = \frac{\cancel{100}}{10\cancel{000}} p(1-p)$$

$$= \frac{1}{100} p(1-p)$$

$$V(\hat{p}_2) = V\left(\frac{x_1 + 4x_{100}}{5}\right) \underset{\downarrow}{=} \frac{V(x_1) + 16\, V(x_{100})}{25}$$

$x_1, \dots x_{100}$ são indep

$$\underset{i.d.}{=} \frac{p(1-p) + 16\, p(1-p)}{25} = \frac{17}{25}\, p(1-p)$$

$$e_R = \frac{V(\hat{p}_2)}{V(\hat{p}_1)} = \frac{\frac{17}{25}\, p(1-p)}{\frac{p(1-p)}{100}} = \frac{100 \cdot 17\, \cancel{p(1-p)}}{25\, \cancel{p(1-p)}}$$

$= 68 > 1$ logo $\hat{p}_1$ é + eficiente que $\hat{p}_2$.

b) $$V(\hat{p}) \geq \frac{[1 + \text{env}'(\hat{p})]^2}{m\, I(p)}$$

Como $\hat{p}$ é cêntrico, env $= 0$

$$V(\hat{p}) \geq \frac{1}{m I(p)}$$

$$I(p) = E\left(\frac{\partial \log f(x)}{\partial p}\right)^2$$

$$f(x) = p^x (1-p)^{1-x}$$

$$\log f(x) = \log\left[p^x (1-p)^{1-x}\right]$$

$$= \log p^x + \log (1-p)^{1-x}$$

$$= x \log p + (1-x) \log (1-p)$$

$$\frac{\partial l}{\partial p} = \frac{x}{p} + (1-x) \times \frac{-1}{1-p}$$

$$= \frac{x}{p} - \frac{1-x}{1-p}$$

$$E\left(\frac{\cancel{X}}{p}-\frac{1-\cancel{X}}{1-p}\right)^2 = E\left[\frac{\cancel{X}(1-p)-(1-\cancel{X})p}{p(1-p)}\right]^2$$

$$= E\left[\frac{\cancel{X}-\cancel{X}p-p+\cancel{X}p}{p(1-p)}\right]^2 = E\left[\frac{X-p}{p(1-p)}\right]^2$$

$$= \frac{E(X-\overset{E(X)}{\overset{\shortparallel}{p}})^2}{[p(1-p)]^2} = \frac{V(X)}{[p(1-p)]^2} = \frac{\cancel{p(1-p)}}{[p(1-p)]^{\cancel{2}}}$$

$$= \frac{1}{p(1-p)} = I(p)$$

$$V(\hat{p}) \geq \frac{1}{m \times \frac{1}{p(1-p)}} = \boxed{\frac{p(1-p)}{m}} > 0$$

var. mín. p/ um estimador qualquer centrado

e) $\hat{p}_1$ é o + eficiente porque $V(\hat{p}_1) = V$mínima

$$\frac{p(1-p)}{m} = \frac{p(1-p)}{100}$$

alínea b)

d) Eficiência absoluta:

$\boxed{0 \leq e_A \leq 1}$

$$e_A = \frac{\frac{1}{mI(p)}}{V(\hat{p})} = \frac{\frac{p(1-p)}{m}}{V(\hat{p})}$$

$$e_{A\,\hat{p}_2} = \frac{\frac{p(1-p)}{100}}{V(\hat{p}_2)} = \frac{\frac{p(1-p)}{100}}{\frac{17}{25}p(1-p)} = \frac{1}{68}$$

(4) $p(x) = \frac{1}{\alpha} e^{-\frac{x}{\alpha}}$ Supor dado: $E(X) = \frac{1}{\beta} = \alpha$

M.M

a) $\overline{X} = E(X) = \alpha \Rightarrow \hat{\alpha} = \overline{X}$

M.M.V.

$$p(x_1, \ldots, x_n \mid \alpha) \underset{\text{indep}}{=} \prod_{i=1}^{n} p(x_i \mid \alpha)$$

$$= \frac{1}{\alpha^n} e^{-\frac{1}{\alpha}\sum x_i} = L$$

$$l = \ln(L) = -n \ln(\alpha) - \frac{1}{\alpha} \sum x_i = 0$$

$$\frac{\partial l}{\partial \alpha} = \frac{-n}{\alpha} - \frac{1}{\alpha^2} \sum x_i$$

$$= \frac{-n\alpha + \sum x_i}{\alpha^2} = 0$$

$$\Leftrightarrow -n\alpha + \sum x_i = 0 \Leftrightarrow \alpha = \frac{\sum x_i}{n} \quad \text{i.e.} \quad \alpha = \overline{X}$$

b) Feito noutras aulas

c) $$P(X \geq 10) = \int_{10}^{+\infty} \frac{1}{\alpha} e^{-\frac{x}{\alpha}} dx = \left[-e^{-x/\alpha}\right]_{10}^{+\infty}$$

$$= 0 + e^{-10/\alpha} = e^{-10/\alpha} = e^{-10/10} = e^{-1} \simeq 0{,}37$$

estimativa:

$$\hat{\alpha} = \bar{x} = \frac{12 + 8 + \ldots + 9}{10} = 10$$

(5) a) M.M

V.A A.A

$$E(x) = p = \overline{X} = \frac{1}{m} \sum x_i \Longleftrightarrow \boxed{\hat{p} = \overline{X}}$$

M.M.V.

$$p(x_1, \dots, x_m \mid p) \underset{ind}{=} \prod_{i=1}^{m} p(x_i \mid p)$$

$$= p^{\sum x_i} (1-p)^{m - \sum x_i} = L$$

$$l = \ln(L) = \ln\left(p^{\sum x_i} (1-p)^{m - \sum x_i}\right)$$

$$= \ln\left(p^{\sum x_i}\right) + \ln\left((1-p)^{m - \sum x_i}\right)$$

$$= \sum x_i \ln p + m - \sum x_i \ln(1-p)$$

$$\frac{\partial l}{\partial p} = \frac{\sum x_i}{p} - \frac{m - \sum x_i}{1-p}$$

$$= \frac{\sum x_i (1-p) - mp + \cancel{\sum x_i p}}{p(1-p)} = \frac{\sum x_i - mp}{p(1-p)}$$

$$\frac{\sum x_i - np}{(1-p)p} = 0$$

$$\sum x_i - np = 0$$

$$p = \frac{\sum x_i}{n} = \overline{x} \Rightarrow \hat{p} = \overline{X}$$

b) <u>consistência</u>

$$\lim_{n\to+\infty} E(\hat{\theta}) = \theta \quad e \quad \lim_{n\to+\infty} V(\hat{\theta}) = 0$$

$$E(\hat{p}) = E\left(\frac{1}{n}\sum x_i\right)$$

$$= \frac{1}{n} E(\sum x_i) \underset{i.d}{\overset{\downarrow}{=}} \frac{1}{n} \sum E(X) = \frac{1}{\cancel{n}} \cdot \cancel{n} \cdot p = p$$

- $\lim_{n\to+\infty} E(\hat{p}) = \lim_{n\to+\infty} p = p \cdot 1 = p$

$$V(\hat{p}) = V\left(\frac{1}{n}\sum x_i\right)$$

$$= \frac{1}{n^2} V(\sum x_i) \underset{i.d.}{\overset{\downarrow}{=}} \frac{1}{n^2} \sum V(X) = \frac{1}{n^{\cancel{2}}} \cdot \cancel{n} \cdot p(1-p)$$

$$= \frac{p(1-p)}{n}$$

- $\lim_{n\to+\infty} \frac{p(1-p)}{n} = 0$ porque são números

<u>É consistente</u>

Eficiência

$$V(\hat{\theta}) \geq \frac{[1 + \text{vies}'(\hat{\theta})]^2}{n I(\theta)}$$

como $\hat{p} = \bar{x}$ é cêntrico logo $\text{vies}(\hat{p}) = 0$

$$I(\hat{p}) = E\left(\frac{\partial \log f(x)}{\partial p}\right)^2$$

$$f(x) = p^x (1-p)^{1-x}$$

$$\log f(x) = \log\left(p^x (1-p)^{1-x}\right)$$

$$= \log(p^x) + \log\left[(1-p)^{1-x}\right]$$

$$= x \log p + (1-x) \log(1-p)$$

$$= \frac{x}{p} + \frac{1-x}{1-p}$$

$$= \frac{x(\cancel{1}-p) + p + \cancel{x}p}{p(1-p)}$$

$$= \frac{x-p}{p(1-p)}$$

$$E\left(\frac{x-p}{p(1-p)}\right)^2$$

$$= \frac{E(x-p)^2}{[p(1-p)]^2} = \frac{V(x)}{[p(1-p)]^2}$$

$$= \frac{p\cancel{(1-p)}}{[p(1-p)]^{\cancel{2}}} = \frac{1}{p(1-p)} = I(p)$$

$$V(\hat{p}) \geq \frac{1}{n\frac{1}{p(1-p)}} = \frac{p(1-p)}{n} = \text{var. min}$$

logo é eficiente

Suficiência → critério da factorização

$$f(x_1, ..., x_n) = \prod_{i=1}^{n} f(x_i)$$

$$= p^{x_1}(1-p)^{1-x_1} \, p^{x_2}(1-p)^{1-x_2} \cdots p^{x_n}(1-p)^{1-x_n}$$

$$= p^{\sum x_i}(1-p)^{\sum(1-x_i)} = p^{\sum x_i}(1-p)^{n-\sum x_i}$$

$$= \underbrace{p^{n\bar{x}}(1-p)^{n-n\bar{x}}}_{g(\hat{p} \mid p)} \cdot \underbrace{1}_{\times h(x_1, ..., x_n)}$$

$\hat{p}$ é suficiente

6 a) M.M

$$E(X) = \alpha \Leftrightarrow \bar{X} = \frac{1}{n}\sum x_i \Leftrightarrow \boxed{\hat{\alpha} = \bar{X}}$$

$$E(X) = \frac{\alpha - \cancel{\sqrt{3}\beta} + \alpha + \cancel{\sqrt{3}\beta}}{2} = \alpha$$

$$V(X) = \frac{(\alpha - \sqrt{3}\beta - (\alpha - \sqrt{3}\beta))^2}{12}$$

$$= \frac{(2\sqrt{3}\beta)^2}{12} = \frac{12\beta^2}{12} = \beta^2$$

$$\beta^2 = V(X) \Leftrightarrow \beta = \sqrt{S^2} \Leftrightarrow \boxed{\hat{\beta} = S}$$

b)

$$\hat{\alpha} = \bar{X} = \frac{3 + 2,5 + 4 + 1,5 + 4}{5} = 3$$

$$\beta = S = \sqrt{S^2} = \sqrt{\frac{(3-3)^2 + (2,5-3)^2 + (4-3)^2 + (1,5-3)^2 + (4-3)^2}{5}}$$

$$= 0,95$$

$\sum p(x_i) = 1$

7 a) $1 - 4\beta + 2\beta + k + \beta = 1$

$-\beta + k = 0$

$\boxed{k = \beta}$

b) M.M. $E(X) = \overbrace{7\beta - 2 = \bar{X}}^{V.A} = \overbrace{\frac{1}{n}\sum x_i}^{A.A}$

$= x_i \cdot p(x_i)$

$E(X) = -2(1-4\beta) + -1(2\beta) + \beta$

$= -2 + 8\beta - 2\beta + \beta$

$= -2 + 7\beta = \bar{X}$

$\Leftrightarrow \boxed{\hat{\beta} = \frac{\bar{X} + 2}{7}}$

$\bar{X}_0 = \frac{0 - 2 - 1 - 1 + 0 + 1 - 2 + 1 - 1 - 1}{10}$

$= -0{,}6$

$\hat{\beta}_0 = \frac{-0{,}6 + 2}{7} = \overset{= \frac{1}{5}}{0{,}2} = P(X=1)$

$E(X^2) = x_i^2 \, p(x_i)$

$= (-2)^2(1-4\beta) + (-1)^2 2\beta + 1^2\beta$

$= 4(1-4\beta) + 2\beta + \beta$

$= 4 - 16\beta + 2\beta + \beta$

$= \boxed{4 - 13\beta}$

$V(X) = E(X^2) - [E(X)]^2$

$= 4 - 13\beta - (-2 + 7\beta)^2$

$= 4 - \frac{13}{5} - (-0{,}6)^2 = 1{,}04$

M.M.V

$$p(x_1, \dots, x_m \mid \beta) \underset{i.i.d}{=} (1-4\beta) \cdot 2\beta \cdot 2\beta$$

$$2^{m_2} \cdot \beta^{m_2}$$

Caso Geral $(1-4\beta)^{m_1} \cdot 2\beta^{m_2} \, \beta^{m_3} \, \beta^{m_4} = L(\beta)$

$$l = \ln(L(\beta)) = m_1 \ln(1-4\beta) + m_2 \ln 2 + (m_2+m_3+m_4)\ln\beta$$

$$\frac{\partial l}{\partial \beta} = m_1 \cdot \frac{-4}{1-4\beta} + (m_2+m_3+m_4) \cdot \frac{1}{\beta}$$

$$\Leftrightarrow -4m_1\beta + (m_2+m_3+m_4)(1-4\beta) = 0$$

$$\Leftrightarrow -4m_1\beta + (m_2+m_3+m_4) - 4\beta(m_2+m_3+m_4) = 0$$

$$\Leftrightarrow -4\beta \underbrace{(m_1+m_2+m_3+m_4)}_{m} + (m_2+m_3+m_4)$$

$$\Leftrightarrow \beta = \frac{m_2+m_3+m_4}{4m} \Leftrightarrow \frac{m-m_1}{4m} = \frac{1}{4} - \frac{m_1}{4m}$$

$$\frac{\partial^2 \ln L(\beta)}{\partial \beta^2} = -4 \times (-1)(1-4\beta)^{-2} \times (-4) + (m_2+m_3+m_4) \times (-1) \cdot \beta^{-2}$$

$$= \frac{16 m_1 \;>0}{(1-4)\beta^2 > 0} - \frac{m_2+m_3+m_4 \geq 0}{\beta^2 > 0} < 0$$

8) a) M.M

$E(X) = \lambda \Leftrightarrow \overline{X} = \frac{1}{m} \sum xi$

$\Leftrightarrow \boxed{\overline{X} = \hat{\lambda}}$

M.M.V

$p(x_1, \ldots x_m | \lambda) = \prod_i \frac{e^{-m\lambda} \cdot \lambda^{\sum xi}}{\prod xi!} = L(\lambda)$

$\ell = \ln(L(\lambda)) = \ln\left(\prod \frac{e^{-m\lambda} \cdot \lambda^{\sum xi}}{\prod xi!}\right)$

$= \ln(e^{-m\lambda}) + \ln(\lambda^{\sum xi}) - \ln(\prod xi!)$

$= -m\lambda \cancel{\ln e} + \sum xi \ln\lambda - \ln(\prod xi!)$

$\frac{\partial \ell}{\partial \lambda} = -m\lambda + \frac{\sum xi}{\lambda} = 0$

$-m\lambda + \sum xi = 0$

$\boxed{\hat{\lambda} = \frac{\sum xi}{m} = \overline{X}}$

b) $\lim_{n \to +\infty} E(\overline{X}) = \lambda$ e $\lim_{n \to +\infty} Var(\overline{X}) = 0$

$E(\overline{x}) = E\left(\frac{1}{m} \sum xi\right) = \frac{1}{m} E(\sum xi) \underset{i.i.d.}{=} \frac{1}{m} \sum \lambda = \lambda$ //

$\lim_{n \to +\infty} \lambda = \lambda \cdot 1 = \lambda$ //

$V(\overline{x}) = V\left(\frac{1}{m} \sum xi\right) = \ldots = \frac{\lambda}{m}$

$$\lim_{n \to +\infty} V(\overline{X}) = \lim_{n \to +\infty} \frac{\lambda}{m} = \lambda \lim_{n \to +\infty} \frac{1}{m}$$

$$= 0$$

Logo é consistente

$$V(\hat{\lambda}) \geq \left[\frac{1 + enu'(\hat{\lambda})}{m\, I(\lambda)} \right]^2$$

$V(\hat{\lambda}) \geq \frac{1}{m\, I(\lambda)}$ pq é cêntrico

$$I(\lambda) = E\left[\left(\frac{\partial \ln p(X \mid \lambda)}{\partial \lambda} \right)^2 \right]$$

$$= E\left(\frac{\partial \log f(x)}{\partial \lambda} \right)^2$$

$$f(x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

$$\log\left(\frac{e^{-\lambda} \cdot \lambda^{x}}{x!} \right)$$

$$= \ln(e^{-\lambda}) + \ln(\lambda^{x}) - \ln(x!)$$

$$= -\lambda \ln e + x \ln \lambda - \ln(x!)$$

$$\frac{\partial \ln p(x)}{\partial \lambda} = -\frac{\lambda}{\lambda} + \frac{x}{\lambda} = -1 + \frac{x}{\lambda}$$

$$I(\lambda) = E\left(-\lambda + \frac{x}{\lambda} \right)^2$$

$$\frac{1}{\lambda^2} E(-\cancel{\lambda} + x)^2 = \frac{1}{\lambda^2} E(-\lambda + x)^2$$

$$= \frac{1}{\lambda^2} V(x)^2 = \frac{\lambda}{\lambda^2} = \frac{1}{\lambda}$$

$$V(\bar{X}) \geq \frac{1}{n \cdot \frac{1}{\lambda}} = \frac{\lambda}{n} \quad \text{var. min}$$

Como $\bar{x}$ tem var. mínima, é o estimador
⊕ eficiente

Suficiência

$$f(x_1, \dots, x_n) = \prod^{n} f(x_i)$$

$$= \frac{e^{-n\lambda} \cdot \lambda^{\sum x_i}}{\prod x_i!}$$

$$= \underbrace{e^{-n\lambda} \cdot \lambda^{\sum x_i}}_{\bar{x}} \cdot \frac{1}{\prod x_i!}$$

$$= \left[e^{-\lambda} \lambda^{\frac{1}{n}\sum x_i}\right]^n \cdot h(x_1, \dots, x_n)$$

É suficiente

9 a) M.M.

$1^\circ M: E(X) = \mu \Leftrightarrow \frac{1}{n} \sum x_i = \bar{x}$

$2^\circ M: E(X^2) = \sigma^2 + \mu^2 \Leftrightarrow \frac{1}{n} \sum x_i^2$

$$\begin{cases} \mu = \bar{x} \\ \sigma^2 + \mu^2 = \frac{1}{n} \sum x_i^2 \end{cases} \quad \begin{cases} \sigma^2 + \bar{x}^2 = \frac{1}{n} \sum x_i^2 \end{cases}$$

$$\begin{cases} \overline{\sigma^2} = \frac{1}{n} \sum x_i^2 - \bar{x}^2 \end{cases} \quad \begin{cases} \overline{\sigma^2} = \frac{1}{n} \sum (x_i - \bar{x})^2 = S^2 \end{cases}$$

$\boxed{\hat{\mu} = \bar{x} \text{ e } \hat{\sigma}^2 = S^2}$ Como tem 2 parâmetros tem de se fazer os 2 MM.

M.M.V

$$f(x_1, \dots, x_m \mid \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\Sigma x_i - \mu}{\sigma}\right)^2}$$

$$f = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{\Sigma x_i - \mu}{\sigma}\right)^2}$$

$$= (\sigma^2 2\pi)^{-\frac{1}{2}} e^{-\frac{1}{2}(\sigma^2)^{-1}(\Sigma x_i - \mu)^2}$$

$$\alpha = (2\pi\sigma^2)^{-\frac{m}{2}} e^{-\frac{1}{2}(\sigma^2)^{-1}\sum_{i=1}^{m}(x_i - \mu)^2}$$

$$\ell = \ln(\alpha) = -\frac{m}{2}\ln(2\pi\sigma^2) - \frac{1}{2}(\sigma^2)^{-1}\Sigma(x_i - \mu)^2$$

$$\frac{\partial \ell}{\partial \mu} = \frac{1}{2}\cdot 2(\sigma^2)^{-1}\left[\sum_{i=1}^{m}(x_i) - m\mu\right] = 0$$

$$\Leftrightarrow \hat{\mu} = +\frac{1}{m}\sum_{i=1}^{m}(x_i) = \bar{X}$$

$$\frac{\partial \ell}{\partial \sigma^2} = -\frac{1}{2}\cdot\frac{2\pi m}{2\pi\sigma^2} + \frac{1}{2}(\sigma^2)^{-2}\Sigma(x_i - \mu)^2$$

$$= \frac{-m\sigma^2 + \Sigma(x_i - \mu)^2}{2(\sigma^2)} = 0$$

$$\Leftrightarrow \sigma^2 = \frac{1}{m}\Sigma(x_i - \mu)^2 = \frac{1}{m}\Sigma(x_i - \bar{X})^2 = S^2$$

$$\boxed{\hat{\mu} = \bar{X}} \text{ e } \boxed{\hat{\sigma}^2 = S^2}$$

$$* \ \bar{X} = \frac{m}{m^2}\Sigma x_i$$

$$\Leftrightarrow \bar{X} = \frac{1}{m}\Sigma x_i$$

ou

$$\alpha = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{m}{2}\left(\frac{\Sigma x_i - \mu}{\sigma}\right)^2}$$

$$\ln(\alpha) = \ln\left(\frac{1}{\sigma\sqrt{2\pi}}\right) - \frac{m}{2\sigma^2}\Sigma(x_i - \mu)^2$$

$$\frac{\partial \ell}{\partial \mu} = -\frac{2m}{2\sigma^2}\Sigma(x_i - \mu) = 0$$

$$\Leftrightarrow m[(\Sigma x_i) - m\mu] = 0$$

$$\Leftrightarrow m\Sigma x_i - m^2\mu = 0 \ *$$

b) $\hat{\mu} = \bar{x} \qquad \hat{\sigma}^2 = S^2$

$$\hat{\mu} = \bar{x} = \frac{7+9+13+8+9+10+14}{7} = 10$$

$$\hat{\sigma}^2 = S^2 = V(X)$$

$$E(X^2) = \sqrt{\frac{(7-10)^2 + (9-10)^2 + (13-10)^2 + (8-10)^2 + (9-10)^2 + (14-10)^2}{7}}$$

$$= \frac{40}{7}$$

c) $E(\hat{\mu}) = E(\bar{X}) = E\left(\frac{1}{n} \Sigma x_i\right)$

$$= \frac{1}{n} E(\Sigma x_i) \underset{i.i.d}{=} \frac{1}{n} \Sigma E(X)$$

$$= \frac{1}{\cancel{n}} \cdot \cancel{n} \cdot \mu = \mu$$

$$\hat{\sigma}^2 = V(\bar{X}) = V\left(\frac{1}{n} \Sigma x_i\right)$$

$$= \frac{1}{n^2} V(\Sigma x_i) \underset{\text{ind}}{=} \frac{1}{n^2} \Sigma V(x_i) \underset{i.i.d}{=} \frac{1}{n^2} \Sigma V(X)$$

$$= \frac{1}{n^{\cancel{2}}} \cancel{n} \cdot \sigma^2 = \frac{1}{n} \sigma^2$$

São cêntricos

d) $f(x_1, \ldots, x_n) = \prod_{i=1}^{n} f(x_i)$

$$= \frac{1}{\sigma^n (\sqrt{2\pi})^n} \times e^{-\frac{1}{2\sigma^2} \Sigma (x_i - \mu)^2}$$

$$= \frac{1}{\sigma^m (\sqrt{2\pi})^m} \cdot e^{-\frac{1}{2\sigma^2}(\sum x_i^2 - 2\mu \underbrace{\sum x_i}_{m\bar{x}} + \underbrace{\sum \mu^2}_{m\mu^2})}$$

$$= \frac{1}{\sigma^m (\sqrt{2\pi})^m} \times e^{-\frac{1}{2\sigma^2}\sum x_i^2} \times e^{-\frac{1}{2\sigma^2}(-2\mu m \bar{x} + m\mu^2)}$$

$$= \underbrace{\frac{1}{\sigma^m (\sqrt{2\pi})^m} \times e^{\frac{m\mu}{\sigma^2}(\bar{x} - \frac{\mu}{2})}}_{g} \times \underbrace{e^{-\frac{1}{2\sigma^2}\sum x_i^2}}_{h(x_1, \dots)}$$

(b) M.M.V p/ estimar θ

$$f(x_1, \dots, x_m) = (1-\theta)^{\sum x_i} \theta^m$$

$$\ell = \ln\left[(1-\theta)^{\sum x_i} \theta^m\right]$$

$$= \ln\left[(1-\theta)^{\sum x_i}\right] + \ln \theta^m$$

$$= \sum x_i \ln(1-\theta) + m \ln\theta$$

$$\frac{\partial \ell}{\partial \theta} = \frac{-\sum x_i}{1-\theta} + \frac{m}{\theta} = 0$$

$$\Leftrightarrow -\sum x_i \cdot \theta + m - m\theta = 0$$

$$\Leftrightarrow \theta(-\sum x_i - m) = -m$$

$$\theta = \frac{-m}{-\sum x_i - m} \qquad \frac{\partial^2 \ell}{\partial \theta^2} < 0$$

$$\theta = \frac{m}{\sum x_i + m}$$

$$\Leftrightarrow \hat{\alpha} = \frac{1}{\theta} = \frac{1}{m}\sum x_i + \cancel{\frac{m}{m}} = \frac{1}{m}\sum x_i + 1$$

$$= \bar{X} + 1$$

$$\hat{\alpha} = \bar{x} + 1 = \frac{0+0+1+0+1}{5} + 1 = \frac{2}{5} + 1 = 1{,}4$$

11 a) M.M.V.

$$p(x_1, \dots, x_n | \theta) = p(x_i)$$

$$= \frac{1}{\theta^n} = L(\theta)$$

$$\ell = \ln\left(\frac{1}{\theta^n}\right) = \ln(\theta^{-n}) = -n \ln \theta$$

$$\frac{\partial \ln}{\partial \theta} = -\frac{n}{\theta^n} = 0$$

$$\underbrace{-n = 0}_{\text{Impossível}}$$

$L(\theta)$ é decrescente c/ θ logo interessa escolher o θ menor valor

$$p(x_1, \dots, x_n | \theta) = \begin{cases} \frac{1}{\theta^n} & \text{se } \forall i \quad 0 < x_i < \theta \\ 0 & \text{no resto} \end{cases}$$

$\hat{\theta}$ = máx veros.

$$\Leftrightarrow \boxed{\hat{\theta} = \text{máx}(x_i)}$$

$$\frac{0+\theta}{2} = \bar{x} \Leftrightarrow \hat{\theta} = 2\bar{x}$$

ou

b) M.M.

$$\theta = E(x) = \int_0^\theta x\, p(x)\, dx$$

$$= \int_0^\theta \frac{x}{\theta}\, dx$$

$$= \frac{1}{\theta}\left[\frac{x^2}{2}\right]_0^\theta = \frac{1}{\theta} \cdot \underbrace{\frac{\theta^2}{2}}_{V.A} = \bar{x} \Leftrightarrow \hat{\theta} = 2\bar{x}$$

$\hat{\theta} = 2\overline{X} = 2\left(\frac{1+5+2+3+6}{5}\right)$

$= 6{,}8$ //

⑫ (12) $V(\overline{x}) = E(\overline{x}^2) - [E(\overline{x})]^2$

$\Leftrightarrow \frac{\theta^2}{n} = E(\overline{x}^2) - \mu^2$

$\Leftrightarrow E(\overline{x}^2) = \frac{\theta^2}{n} + \mu^2 \neq \mu^2$

Falsa //

(13) M.M.V

$f(x_1, \dots, x_n \mid \theta) = \prod f(x_i)$

$= e^{-\sum(x_i - \theta)} = L(\theta)$

$l = \ln\left(e^{-\sum(x_i - \theta)}\right)$

$= -\sum(x_i - \theta) \ln e$

$\frac{\partial \ln}{\partial \theta} = -\sum(x_i - \theta) = -\sum x_i + n\theta = 0$

$\Leftrightarrow \theta = \frac{\sum x_i}{n} = \overline{X}$

$\hat{\theta} = \min x_i$

(14) a) M.M.V

$$f(x) = \frac{1}{1-\alpha}$$

$$L(\alpha) = f(x_1, \dots, x_n \mid \alpha) = \prod_{i=1}^{n} f(x_i \mid \alpha)$$

$$= \prod_{i=1}^{n} \frac{1}{1-\alpha} = \left(\frac{1}{1-\alpha}\right)^n$$

$$\ell = \ln\left[\left(\frac{1}{1-\alpha}\right)^n\right] \qquad \frac{\partial \ell}{\partial \alpha} = \frac{-n}{1-\alpha} = 0$$

$$= \ln\left[(1-\alpha)^{-n}\right]$$

$$-n = 0 \Rightarrow \boxed{n = 0}$$

Impossível

$$= -n \ln(1-\alpha)$$

$L(\alpha)$ é crescente c/ α $\Rightarrow$ interessa escolher o maior valor de α. $\Longrightarrow$ Logo $\boxed{\hat{\alpha}_1 = \min x_i}$

b) b1) $\hat{\alpha}_2 = \min x_i$

$$\Downarrow$$

$$\boxed{\hat{\alpha}_2 = 0}$$

b2) $E(\hat{\alpha}) = E(2\overline{X} - 1)$

$= E(2\overline{X}) - 1$

$= 2 \cdot E(\overline{X}) - 1$

$= \cancel{2}\left(\frac{\alpha + 1}{\cancel{2}}\right) - 1 = \alpha$

C.A

$\alpha = 2\overline{X} - 1$

$\overline{X} = \frac{\alpha + 1}{2}$

b3) $\longrightarrow$ não sai

15 a)

$$f(x,\beta)\begin{cases}\frac{1}{2\beta}e^{\frac{x}{\beta}} & x<0\\ \frac{1}{2\beta}e^{-\frac{x}{\beta}} & x>0\end{cases}$$

$$E(X)=\int_{-\infty}^{+\infty} x\, f(x)\, dx$$

$$=\int_{-\infty}^{0}\frac{1}{2\beta}\, x\, e^{x/\beta}\, dx+\int_{0}^{+\infty}\frac{1}{2\beta}\, x\, e^{-x/\beta}\, dx$$

$$E(X^2)=\int_{-\infty}^{+\infty} x^2\, f(x)\, dx$$

17) a) $f(x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$

$P(X=0) = \frac{e^{-\lambda}\lambda^{0}}{x!} = \frac{e^{-\lambda}}{0!} = \boxed{e^{-\lambda}}$, $E(X)=\lambda$

b) $E(\hat{p}) = E\left(\frac{K}{m}\right) = \frac{1}{m} E(K) = \frac{\cancel{m}p}{\cancel{m}} = p = e^{-\lambda} = P(X=0)$

$\Downarrow$

É centrado

$\lim_{n \to +\infty} E(\hat{p}) = \lim_{n \to +\infty} p = p$

$\lim_{n \to +\infty} V(\hat{p}) = \lim_{n \to +\infty} \frac{p(1-p)}{m} = p(1-p) \lim_{n \to +\infty} \frac{1}{m}$

$= 0$

$V(\hat{p}) = V\left(\frac{K}{m}\right) = \frac{1}{m^2} V(K) = \frac{\cancel{m}p(1-p)}{m^{\cancel{2}}} = \frac{p(1-p)}{m}$

$\Downarrow$

É consistente

16) a)

x_i	-1	0	1
$p(x_i)$	$\frac{a\theta}{2}$	$a(1-\theta)$	$\frac{a\theta}{2}$

$\sum p(x_i) = 1 \Leftrightarrow \boxed{a=1}$

M.M

$E(X) = \overline{X}$

$E(X) = -\frac{\theta}{2} + \frac{\theta}{2} = 0$

Logo $E(X) = \overline{X} \Leftrightarrow \boxed{0 = \overline{X}}$

$E(X^2) = \frac{1}{m}\sum_{i=1} x_i^2 = \frac{\theta}{2} + 0 + \frac{\theta}{2} = \frac{\cancel{2}\theta}{\cancel{2}} = \theta$

$\Leftrightarrow \boxed{\hat{\theta} = \frac{1}{m}\sum_{i=1}^{m} x_i^2}$

b) $E(\hat{\theta}) = E\left(\frac{1}{n} \sum x_i^2\right)$

$= \frac{1}{n} \sum E(x_i^2) \underset{\substack{\downarrow \\ i.d \\ X = X_i}}{=} \frac{1}{n} \sum E(X^2) = \frac{1}{\cancel{n}} \cdot \cancel{n} \cdot \theta^2 = \theta^2$

É cêntrico

$\lim_{n \to +\infty} E(X) = \lim_{n \to +\infty} \theta^2 = \theta^2$

É consistente

c) $p(x_1, \dots, x_n / \theta) = a\left(\frac{\theta}{2}\right)^{\sum x_i} \cdot (1-\theta)^{n - \sum x_i} = L(\theta)$

$\ln(L) = \ln a + \sum x_i \ln\left(\frac{\theta}{2}\right) + (n - \sum x_i) \ln(1-\theta)$

$\frac{\partial \ln}{\partial \theta} = \frac{1}{2} \cdot \frac{\sum x_i}{\left(\frac{\theta}{2}\right)} - \frac{n - \sum x_i}{1-\theta} = 0$

$\frac{1}{\cancel{2}} \cdot \frac{\cancel{2} \sum x_i}{\theta} - \frac{n + \sum x_i}{1-\theta} = 0$

$\sum x_i - \theta \cancel{\sum x_i} - n\theta + \theta \cancel{\sum x_i} = 0$

$\theta = \frac{\sum x_i}{n} = \boxed{\overline{X}_n = 0,4}$

$\overline{X} = \frac{2}{5} = 0,4$

0,4

(18) a) $S'^{2} = 20 - \sigma_2$

$IC = 90\%$ $\quad IC: [240, 826{,}25]$

$$\frac{(n-1)S'^2}{\sigma^2} \sim \chi^2_{n-1} \rightarrow \mu \text{ desconhecido}$$

Intervalo probabilidade:

$$P\left(a \leq \frac{(n-1)S'^2}{\sigma^2} \leq b\right) = 90\%$$

Intervalo confiança p/ σ^2

$$\frac{(n-1)S'^2}{\sigma^2} \leq b \rightarrow \text{Qui-quadrado só tem valores positivos}$$

$$\Leftrightarrow (n-1)S'^2 \leq b\sigma^2$$

$$\left[\frac{(n-1)S'^2}{b}, \frac{(n-1)S'^2}{a}\right]$$

$$[240 ; 826{,}45]$$

Sabemos $S'^2 = 20^2 = 400$

Dado $\sqrt{S'^2} = 20$

$$\begin{cases} \frac{(n-1)400}{b} = 240 \\ \frac{(n-1)400}{a} = 826{,}45 \end{cases}$$

$n = 1 + b \cdot \frac{240}{400} = 1 + 0{,}6b$ $\quad b \sim 95\%$

$n = 1 + a\,\frac{826{,}45}{400} = 1 + 2{,}065a$ $\quad a \sim 5\%$

χ^2_{n-1}

χ^2_{12}

$(n = 13) \Rightarrow a_{(5\%)} = 5{,}23$

$1 + 2{,}065 \times 5{,}23 = 11{,}9$

$\boxed{n = 16} = a = 7{,}26 = 1 + 2{,}065 \times 7{,}26 = 15{,}99 \sim 16$

χ^2_{15}

(18) b) $[196{,}08 ; 1147{,}03]$ $CI = ?$

$$\begin{cases} \dfrac{(n-1)\,400}{b} = 196{,}08 \\ \dfrac{(n-1)\,400}{a} = 1147{,}03 \end{cases} \quad b = \frac{(16-1)\,400}{196{,}08} \cong 30{,}6 \qquad a = \frac{(16-1)\cdot 400}{1147{,}03} = 5{,}23$$

Tabela χ^2_{15} procurar $30{,}6 \Rightarrow 0{,}99$//

" " $5{,}23 \Rightarrow 0{,}01$//

$IC = 0{,}99 - 0{,}01 = 0{,}98 = 98\%$//

c) $n = 169 \Longrightarrow 101$ desacordam

Probabilidade p/ a proporção

$$P\left(-c \leq \frac{\bar{x} - p}{\sqrt{\frac{\bar{x}(1-\bar{x})}{n}}} \leq c\right) = 95\%$$

$(n \geq 100)$

$$\hat{p} = \bar{x} = \frac{1}{169} \sum_{i=1}^{169} x_i = \frac{101}{169} \cong 0{,}60 \;(60\%)$$

$$P\left(-c \leq \frac{\bar{x} - p}{0{,}0377} \leq c\right) = 95\% \rightarrow N(0,1) \rightarrow \text{Tabela D}$$

$\downarrow$ 95% $\downarrow$ $c(D) = 1{,}96$

$p\,[\bar{x} - 0{,}377 \cdot c \;;\; \bar{x} + 0{,}0377 \cdot c]$

$[0{,}598 - 0{,}0377 \times 1{,}96 \;;\; 0{,}598 + 0{,}377 \times 1{,}96]$

(19) a) $n = 1200$//

$IC = 90\% \Rightarrow$ p/ a proporção

240 desconheciam

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{1200} x_i = \frac{960}{1200} = 0{,}8$$

$$P\left(-c \leq \frac{\bar{x} - p}{\sqrt{\frac{\bar{x}(1-\bar{x})}{n}}} \leq c\right) = 90\%$$

$$P\left(-c \leq \frac{\overline{x} - p}{0{,}01155} \leq c\right) = 90\% \quad \sim N(0,1) \to 90\%$$

$$\Phi(D) = 1{,}645 \to c$$

$IC\ [\overline{x} - 0{,}01155c \,;\ \overline{x} + 0{,}01155c]$

$IC\ [0{,}781 \,;\ 0{,}819]$

b) $[0.767 \,;\ 0.833]$

$0{,}8 - 0{,}01155c = 0.767$

$\Leftrightarrow \boxed{c = 2{,}857}$ ✓

$\Phi(2.86) = 99.58\%$ //

20 a) $\overline{x} = 2400\text{ kg}$ $\quad n = 40$

$\sqrt{S^2} = 150\text{ kg} \Rightarrow S'^2 = \frac{n}{n-1} S^2 \to S' = \sqrt{\frac{40}{39}}\, S$

$= 1{,}013 \times 150 = 152$ //

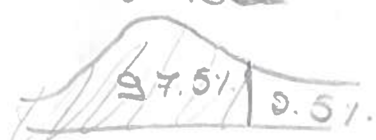
$X_i \sim N(\mu, \sigma^2)$

σ^2 conhecido

$$P\left(-c \leq \frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \leq c\right)$$

$$P\left(-c \leq \frac{\overline{x} - \mu}{\frac{150}{\sqrt{40}}} \leq c\right)$$

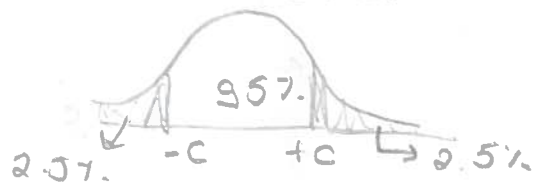
Temos Tabela



$\mu \in [\overline{x} - c \cdot \frac{150}{\sqrt{40}} \,;\ \overline{x} + c\,\frac{150}{\sqrt{40}}]$

$\overline{x} \to 2400$

Queremos



p/ IC = 95% → c = 2.02

$[2352, 2448]$

p/ IC = 99% → c = 2.70

$[2336, 2464]$

aumentou 10 → Tabela T-Student

0.975	30	40
	2.04	2.02

diminuiu $\boxed{-2.02}$

10 —— −0,02

9 —— x ⟶ = −0,018

(aumentou desde 30, i.e. 2,04)

valor: 2,04 − 0,018

$\boxed{2{,}022 = c}$

b) $\bar{x} \pm 35$

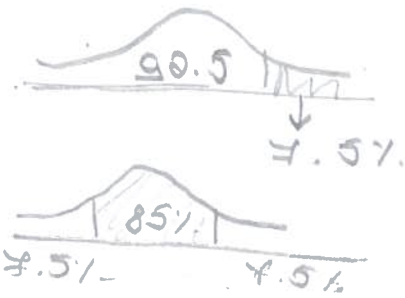
$$c \cdot \frac{s'}{\sqrt{n}} = 35$$

$$c = \frac{35 \cdot \sqrt{40}}{150} = 1.48$$

0.9 0.925 0.95

1.30 1.68

meio 1.49



R.: 85%

21 → não sai

22 a) A > B n = 1000 → x_i A = 450

x_i B = 550

F. Bernoulli → $X \sim Bi(1, p)$

M.M.V $\quad L(p) = p^{\sum x_i}(1-p)^{n - \sum x_i}$

$$\ln(L(p)) = \ln(p^{\sum x_i}) + \ln\left[(1-p)^{n-\sum x_i}\right]$$

$$= \sum x_i \ln(p) + n - \sum x_i \ln(1-p)$$

$$\frac{\partial \ell}{\partial p} = \frac{\sum x_i}{p} + \frac{n - \sum x_i}{1-p} = 0$$

b

$$(1-p)\sum x_i - pn + p\sum x_i = 0$$

$$\sum x_i - pn = 0$$

$$p = \frac{1}{n}\sum x_i \rightarrow \boxed{\hat{p} = \bar{X}}$$

$$\hat{p} = \frac{450}{1000} = 0{,}45$$

b) $P\left(-c \leq \dfrac{\bar{X} - p}{\sqrt{\dfrac{0{,}45(1-0{,}45)}{1000}}} \leq c\right) = 95\%$

$[\bar{x} - c \cdot 0{,}0157 \; ; \; \bar{x} + c \cdot 0{,}0157] = 95\%$

$\bar{x} \downarrow 0.45$

ver na tabela D do [illegible]

$\boxed{c = 1.960}$

$[0{,}419 \; ; \; 0{,}481]$ ✓

23) $n = 400 \longrightarrow x_i = 340$

$\bar{x} = \dfrac{340}{400} = 0{,}85$

$0{,}8 \leq p \leq 0{,}9$

$P\left(-c \leq \dfrac{\bar{X} - p}{0{,}018} \leq c\right) = 95\%$

$[0{,}85 - c \cdot 0{,}018 \; ; \; 0{,}85 + c \cdot 0{,}018] = 95\%$

$\boxed{c = 1.960}$

$[0{,}81472 \; ; \; 0{,}88528] \in [0{,}8 \; ; \; 0{,}9]$

Sim //

24) a) $IC = 90\%$

$\bar{x} = \dfrac{30}{400} = 0.075$

$P\left(-c \leq \dfrac{\bar{X} - p}{\sqrt{\dfrac{\bar{x}(1-\bar{x})}{n}}} \leq c\right) = 90\%$

$[\bar{x} - c \cdot 0{,}0132 \; ; \; \bar{x} + c \cdot 0{,}0132] = 90\% \longrightarrow \boxed{c = 1.645}$

amplitude: $2c\sqrt{\frac{\bar{x}(1-\bar{x})}{400}} = 0.0433 > 0.04$

R.: Não

1ª solução: $p = 1/2$

2ª solução: usar $p = \bar{x} = 7.5\%$

$Var(\bar{x}) = \frac{(1-\bar{x})\bar{x}}{n} = \frac{1/4}{400} \longrightarrow$ pior caso

1º $2c\sqrt{\frac{1/2(1-1/2)}{n}} = 0.04$ e $c = 1.645$

2º $2c\sqrt{\frac{\frac{3}{40}(1-\frac{3}{40})}{n}} = 0.04$ e $c = 1.645$

1º $\left(\frac{c}{0.02}\right)^2 1/2(1-1/2) = 1691 \Rightarrow \boxed{n = 1692}$

2º $\left(\frac{c}{0.02}\right)^2 \cdot \frac{3}{40}\left(1-\frac{3}{40}\right) = 468...$ logo $n = 470$

28) a) M.M

$V(X) = \frac{2}{\theta}$

1º M.O: $E(X) = \mu = \bar{X} = \frac{1}{n}\sum x_i$

2º M.O: $E(X^2) = \mu^2 + \sigma^2 = \frac{1}{n}\sum x_i^2$

$V(X) = E(X^2) - [E(X)]^2$

$\frac{2}{\theta} = \mu^2 + \frac{2}{\theta} - \mu^2$

b) $\hat{\alpha} = \bar{x} - \frac{4}{3}$

$E(\hat{\alpha}) = E\left(\bar{x} - \frac{4}{3}\right) = E(\bar{x}) - \frac{4}{3} = \alpha + \cancel{\frac{4}{3}} - \cancel{\frac{4}{3}}$

sim//

c) $n = 200$ // $\quad \sigma = \frac{2}{9} \quad \sigma'^2 = \sqrt{\frac{2}{9} \times \frac{200}{(200-1)}} = 0{,}4726$

$[9.934, \ 10.066]$

$P\left(-c \leq \frac{\bar{X} - \mu}{\frac{S'}{\sqrt{n}}} \leq c\right) = 99\% \longrightarrow t(n-1)$

$\boxed{c = 2.576}$

$\begin{cases} \bar{x} - c \cdot \frac{0{,}4726}{10\sqrt{2}} = 9.934 \\ \bar{x} + c \cdot 0.0334 = 10.066 \end{cases} \quad \begin{cases} \bar{x} = 10.02 \\ \bar{x} = 9{,}98 \end{cases} \quad \bar{x} \simeq 10$ //

d) $n = ?$

amplitude : $2c \cdot \frac{S'}{\sqrt{n}} = b - a$

$= 10.066 - 9.934 = 0.132$

$2 \times 2.576 \cdot \frac{0.4726}{\sqrt{n}} = 0.132$

$\Leftrightarrow \boxed{n \simeq 340}$

29 a) $n = 1600$ // $\quad P = 99\% \Rightarrow \boxed{c = 2.576}$

$2c \cdot \sqrt{\frac{\bar{x}(1-\bar{x})}{n}} = \text{amp}$

$2 \cdot 2.576 \sqrt{\frac{\bar{x}(1-\bar{x})}{1600}} = \text{amp}$

b) $\bar{X} = 10\%$

$$\left(-c < \frac{\bar{x}-p}{\underbrace{\sqrt{\frac{\bar{x}(1-\bar{x})}{n}}}_{\frac{3}{400}}} < c\right) = 99\% \Rightarrow \boxed{c = 2.576}$$

$$\left[0{,}1 - 2.576 \cdot \frac{3}{400} \; ; \; 0{,}1 + 2.576 \cdot \frac{3}{400}\right]$$

$$[0.08068 ; \; 0{,}11932]$$

30)

$$EQM(S) = \frac{V(\hat{\theta}_1) + \text{enb}^2(\hat{\theta})}{V(\hat{\theta}_2) + \text{enb}^2(\theta)}$$

$$E(S) = E\left(\frac{1}{2}X_1 + \frac{1}{3}X_2 + \frac{1}{4}X_3\right)$$

$$= \frac{1}{2}E(X_1) + \frac{1}{3}E(X_2) + \frac{1}{4}E(X_3)$$

$$= \frac{1}{2}\mu + \frac{1}{3}\mu + \frac{1}{4}\mu = \frac{13}{12}\mu \neq \mu$$

i.d. logo $E(X) = \mu$

S é um estimador enviesado

$$E(T) = E\left(\frac{1}{9}X_1 + \frac{1}{9}X_2 + \frac{7}{9}X_3\right) \ldots = \mu$$

T é um estimador centrado

$$E(U) = E\left(\frac{2}{3}X_1 + \frac{1}{9}X_2 + \frac{1}{8}X_3\right) = \ldots = \frac{65}{72}\mu \neq \mu$$

U é um estimador enviesado

$$\text{enb}(S) = E(S) - \mu = \frac{13}{12}\mu - \mu = \frac{1}{12}\mu$$

$$\text{enb}(U) = \ldots = -\frac{7}{72}\mu$$

$$V(S) = \frac{1}{2^2}V(X_1) + \frac{1}{3^2}V(X_2) + \frac{1}{4^2}V(X_3) = \frac{61}{144}\sigma^2$$

$$V(U) = \left(\frac{2}{3}\right)^2 \sigma^2 + \frac{1}{9^2}\sigma^2 + \frac{1}{8^2}\sigma^2 = \frac{2449}{5184}\sigma^2$$

$$EQM(S) = \frac{61}{144}\sigma^2 + \left(\frac{1}{12}\right)^2 \mu^2 = 0{,}4236\,\sigma^2 + 0{,}0069\,\mu^2$$

$$EQM(T) = \mu^2$$

$$EQM(U) = \left(\frac{65}{72}\right)^2 \mu^2 + \frac{2449}{5184}\sigma^2 = 0{,}47245\,\sigma^2 + 0{,}815\,\mu^2$$

$$eR = \frac{EQM(S)}{EQM(T)} = \frac{0{,}4236\,\sigma^2 + 0{,}0069\,\mu^2}{\mu^2} \Rightarrow S \text{ é } + \text{ eficiente que } T$$

$$eR = \frac{EQM(S)}{EQM(U)} = \frac{0{,}42360\,\sigma^2 + 0{,}069\,\mu^2}{0{,}472450\,\sigma^2 + 0{,}815\,\mu^2} \Rightarrow S \text{ é } + \text{ eficiente que } U$$

$$eR = \frac{EQM(T)}{EQM(U)} = \frac{\mu^2}{0{,}472450\,\sigma^2 + 0{,}815\,\mu^2} \Rightarrow U \text{ é } + \text{ ef que } T$$

$$S < U < T$$