Sebenta Estatística I

SOMOS PARA O TEU SUCESSO

COMISSÃO DE 2º ANO 2016/2017

Este é um trabalho realizado por alunos, pelo que não está livre de conter gralhas ou falta de informação; torna-se, assim, essencial fazer uma análise crítica à sua leitura, tendo em conta a matéria lecionada nas aulas. Qualquer correção deverá ser enviada para comissao2ano@aefep.pt

Exercícios de Probabilidade

1

a) $A^n_p = \frac{n!}{(n-p)!} = \frac{10!}{(10-2)!} = 90$

b) $\alpha^n_p = n^p = 6^3 = 216$

c) $C^n_p = \frac{n!}{p!(n-p)!} = \frac{5!}{2!(5-2)!} = 10$

d) $A^5_5 = \frac{5!}{0!} = 120$ ou $P_5 = 5! = 120$

e) $P^{12}_{3,4,5} = \frac{12!}{3!\,4!\,5!} = 27.720$

2

a) $\alpha^6_3 = 6^3 = 216$ elementos

b) $A = \{6,6,6\}$

$B = \{(6,6,6),(6,6,5),(6,5,6),(5,6,6),(5,5,6),(5,6,5),(6,5,5),(6,6,4),(6,4,6),(4,6,6)\}$

$C = \{(6,6,6)\}$

c) $P(A) = \frac{\text{nº casos favoráveis}}{\text{nº casos possíveis}} = \frac{1}{216}$

$P(B) = \frac{10}{216} = \frac{5}{108}$

$P(C) = \frac{1}{216}$

3 R = "saída de rei"

a) C = "saída de copas"

$P(R) = \frac{4}{52} = \frac{1}{13}$

b) $P(C) = \frac{13}{52} = \frac{1}{4}$

c) $P(R \cup C) = P(R) + P(C) - P(R \cap C) = \frac{1}{13} + \frac{1}{4} - \frac{1}{52} = \frac{4}{13}$

d) $P(R\cap\bar{C}) = P(R\setminus C) = P(R) - P(R\cap C) = \frac{1}{13} - \frac{1}{52} = \frac{3}{52}$

e) $P(\bar{R}) = 1 - P(R) = 1 - \frac{1}{13} = \frac{12}{13}$

f) $P(\bar{R}\cap\bar{C}) = P(\overline{R\cup C}) = 1 - P(R\cup C) = 1 - \frac{4}{13} = \frac{9}{13}$

g) $P(\bar{R}\cup\bar{C}) = P(\overline{R\cap C}) = 1 - P(R\cap C) = 1 - \frac{1}{52} = 51/52$

4

a) P("ganhar o 1º premio") $= \frac{5}{100} = 0{,}05$

ganhar 4 em 6 premios

b) P("ganhar 4 premios") $= \frac{C^{6}_{4} \times C^{94}_{1}}{C^{100}_{5}} = 1{,}87\times10^{-5}$

em 100 compra 5

não ganhar nenhum premio

c) P("ganhar pelo menos 1 premio") $= 1 - \frac{C^{6}_{0}\, C^{94}_{5}}{C^{100}_{5}} = 0{,}27$

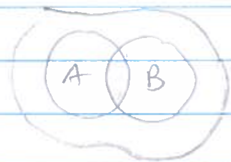
d) P("ganhar pelo menos 2 premios") $= 1 - \left(\frac{C^{6}_{0}\, C^{94}_{5}}{C^{100}_{5}} + \frac{C^{6}_{1}\, C^{94}_{4}}{C^{100}_{5}}\right) = 0{,}028$

5

$P(A) = 0{,}20$

$P(B\setminus A) = 0{,}15$

$P(A\cap B) = 0{,}15$



a) $P(B\setminus A) = P(B) - P(A\cap B)$

$0{,}15 = P(B) - 0{,}15$

$P(B) = 0{,}3 \times 100\% = 30\%$

b) $P(A\setminus B) = P(A) - P(A\cap B) = 0{,}20 - 0{,}15 = 0{,}5 \rightarrow 50\%$

c) $P(A\cup B) = P(A) + P(B) - P(A\cap B) = 0{,}2 + 0{,}3 - 0{,}15 = 0{,}35 \rightarrow 35\%$

d) $P(\bar{A}\cap\bar{B}) = P(\overline{A\cup B}) = 1 - P(A\cup B) = 1 - 0{,}35 = 0{,}65 \rightarrow 65\%$

e) $P(\bar{A}\cup\bar{B}) = 1 - P(A\cap B) = 1 - 0{,}15 = 0{,}85 \rightarrow 85\%$

6. I = "possuir departamento de investigação"
L = "realizar lucros"

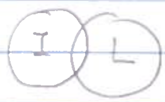
$P(I) = 0,25$
$P(L) = 0,50$
$P(I \cap L) = 0,20$

a) $P(I \cup L) = P(I) + P(L) - P(I \cap L) = 0,25 + 0,50 - 0,20 = 0,55 \rightarrow 0,55\%$

b) $P(I \setminus L) = P(I) - P(I \cap L) = 0,05 \rightarrow 5\%$
$P(L \setminus I) = P(L) - P(I \cap L) = 0,50 - 0,20 = 0,30 \rightarrow 30\%$

$P = 5\% + 30\% = 35\%$

c) $P(\overline{I \cup L}) = 1 - P(I \cup L) = 1 - 0,55 = 0,45 \rightarrow 45\%$

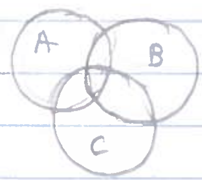


d) $P(I \setminus L) = P(I) - P(I \cap L) = 0,25 - 0,20 = 0,05 \rightarrow 5\%$

e) $P(\bar{I} \cap L) = P(L) - P(I \cap L) = 0,50 - 0,20 = 0,30 \rightarrow 30\%$

7. A → "ler jornal A" $P(A) = 0,098$ $P(A \cap B) = 0,051$
B – "ler jornal B" $P(B) = 0,229$ $P(A \cap C) = 0,037$
C – "ler jornal C" $P(C) = 0,121$ $P(B \cap C) = 0,06$
$P(A \cap B \cap C) = 0,024$

a)
$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$
$= 0,098 + 0,229 + 0,121 - 0,051 - 0,037 - 0,06 + 0,024$
$= 0,324$

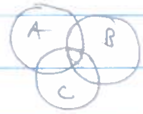


b) $P(A \cap B \cap \bar{C}) = P(A \cap B \setminus C) = P(A \cap B) - P(A \cap B \cap C)$
$= 0,051 - 0,024$
$= 0,027 \rightarrow 2,7\%$

c) $P(A\cap\bar{B}\cap\bar{C}) = P(A) - P(A\cap B) - P(A\cap C) + P(A\cap B\cap C)$

$= 0{,}098 - 0{,}051 - 0{,}037 + 0{,}024$

$= 0{,}034 \rightarrow 3{,}4\%$



8) A e B saõ independentes se a realização de A não afeta a probabilidade de ocorrência de B e vice-versa. Assim:

$P(A|B) = P(A)$ e $P(B|A) = P(B)$

$P(B\setminus A) = P(B) - P(A\cap B)$

$= P(B) - P(A).P(B)$

$= P(B)(1 - P(A))$

$= P(B).P(\bar{A})$

9) $P(A|B) = P(A)$

$P(A\cap B) = P(A).P(B) = 0.3\times 0.2 = 0{,}06$

$P(A\cap B\cap C) = P(B\cap C) = \emptyset$

$P(A) = 0.3$

$P(\bar{B}) = 0.8 = 1 - P(B) \Leftrightarrow P(B) = 0{,}2$

$P(A\setminus C) = 0.1 = P(A) - P(A\cap C)$

$P(A\cup B\cup C) = 0.9$

$P(A\cup C) = P(A) + P(C) - P(A\cap C)$

$= P(A) + P(C) - P(A\cap C)$

$= 0.3 + 0{,}66 - 0.2$

$\boxed{P(A\cup C) = 0{,}76}$

$\downarrow$

76%

$P(A\setminus C) = P(A) - P(A\cap C)$

$P(A\cap C) = P(A) - P(A\setminus C)$

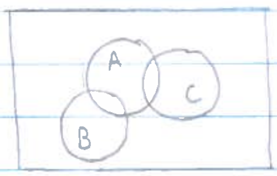
$P(A\cap C) = 0.3 - 0.1$

$= 0.2$

$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(A\cap C) - P(B\cap C) + P(A\cap B\cap C)$

$0.9 = 0.3 + 0.2 + P(C) - 0{,}06 - 0.2 - 0 - 0$

$P(C) = 0{,}66$



$P(B\setminus\bar{C}) = P(\bar{B}) - P(\bar{B}\cap\bar{C})$

$= P(\bar{B}) - P(\overline{B\cup C})$

$= P(\bar{B}) - (1 - P(B\cup C))$

$= 0{,}8 - (1 - P(B) - P(C) + P(B\cap C))$

$= 0{,}8 - (1 - 0{,}2 - 0{,}66 + 0)$

$\boxed{P(B\setminus\bar{C}) = 0{,}66}$

$\downarrow$

66%

(10)

A^n_p

a) $P(A) = \dfrac{C^{13}_1 \times C^{13}_1 \times C^{13}_1}{\alpha^{52}_3} = \dfrac{1}{64}$

b) $P(A) = \dfrac{C^{13}_1 \times C^{13}_1 \times C^{13}_1}{A^{52}_3} = \dfrac{169}{10\,200}$

c) $P(A) = \dfrac{C^{13}_1 \times C^{13}_1 \times C^{13}_1}{C^{52}_3} = \dfrac{169}{1700}$

(11)

a) $P = \dfrac{C^5_1 \times C^5_1}{C^{10}_2} = \dfrac{5}{9}$

b) $P = \dfrac{A^5_1 \times A^5_1}{A^{10}_2} = \dfrac{5}{9}$

c) $P = \dfrac{\alpha^5_1 \times \alpha^5_1 \times A^2_1}{\alpha^{10}_2} = \dfrac{1}{2}$

(12) $A = \{(2,1), (2,2), (2,3), \dots, (2,6), (4,1), \dots\}$

$B = \{(1,1), (2,1), \dots, (6,1), (5,5), (1,3), \dots\}$

$C = \{(2,2), (1,3), (3,1), (3,5), (5,3), (4,4), (6,2), (2,6), (6,6)\}$

a) A e B são independentes se e só se $P(A \cap B) = P(A).P(B)$

$P(A) = \dfrac{A^3_1}{A^6_1} \times \dfrac{A^6_1}{A^6_1} = \dfrac{3}{6} = \dfrac{1}{2}$

$P(B) = \dfrac{A^6_1 \times A^3_1}{A^6_1 \times A^6_1} = \dfrac{1}{2}$

$P(A \cap C) = \dfrac{\cancel{4}\ 5}{A^6_1 \times A^6_1} = \dfrac{5}{36}$

$P(A \cap B) = \dfrac{A^3_1 \times A^3_1}{A^6_1 \times A^6_1} = \dfrac{9}{36} = \dfrac{1}{4}$

$P(C) = \dfrac{9}{A^6_1 \times A^6_1} = \dfrac{1}{4}$

$P(A).P(B) = \dfrac{1}{4} = P(A \cap B)$ $\boxed{\text{SIM}}$

$P(A).P(C) = \dfrac{1}{4} \times \dfrac{1}{2} = \dfrac{1}{8} \neq P(A \cap C) = \dfrac{5}{36}$ $\boxed{\text{NÃO}}$

$P(B).P(C) = \dfrac{1}{2} \times \dfrac{1}{4} = \dfrac{1}{8} \neq P(B \cap C) = \dfrac{1}{9}$ $\boxed{\text{NÃO}}$

b) $\emptyset = P(A \cap B \cap C) \neq P(A) \times P(B) \times P(C) = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{36}$

13

A – "sair pelo menos 1 vez doble 6 em 24 lançamentos"

S_i – "sair doble 6 no lançamento i" $\qquad P(S_1) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

si independente

$$P(A) = 1 - P(\bar{A}) = 1 - P(\bar{S}_1 \cap \bar{S}_2 \cap \dots \cap \bar{S}_{24}) = 1 - P(\bar{S}_1) \times P(\bar{S}_2) \times \dots \times P(\bar{S}_{24}) =$$
$$= 1 - (P(\bar{S}_1))^{24} = 1 - (1 - P(S_1))^{24} =$$
$$= 1 - \left(1 - \left(\frac{1}{36}\right)\right)^{24}$$
$$= 1 - \left(\frac{35}{36}\right)^{24}$$
$$= 0{,}49$$

∴ Não é indiferente apostar a favor ou contra

Deveríamos apostar contra pois $P(\bar{A}) = 1 - P(A) = 1 - 0{,}49 = 0{,}51 > 0{,}49 = P(A)$

14

faces

$P(F) = \frac{2}{3}$

$P(C) = \frac{1}{3}$

coroas

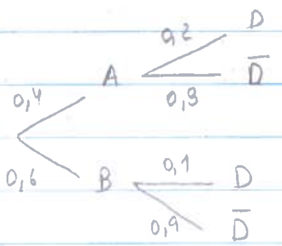
$$P(\text{"ser nº par"}) = \frac{2}{3} \times \frac{{}^{4}A_1}{{}^{9}A_1} + \frac{1}{3} \times \frac{{}^{2}A_1}{{}^{5}A_1}$$
$$= \frac{59}{135} = 0{,}4296 \rightarrow \boxed{42{,}96\%}$$

15

a) A – "a caixa vir da linha A" $\qquad P(A) = 0{,}4 \qquad P(A|B) = \frac{P(A \cap B)}{P(B)}$

B – "a caixa vir da linha B" $\qquad P(B) = 0{,}6$

C_i – "a caixa tem i defeituosos", com i = 1, 2, 3



$$P(C_2) = P(C_2 \cap A) + P(C_2 \cap B)$$
$$= P(C_2|A) \times P(A) + P(C_2|B) \times P(B)$$
$$= C_2^3 \times 0{,}2 \times 0{,}2 \times (1 - 0{,}2) \times 0{,}4 + C_2^3 \times 0{,}1^2 \times (1 - 0{,}1) \times 0{,}6$$
$$= 0{,}0549$$

b) $P(C_0) = P(C_0|A)\,P(A) + P(C_0|B)\,P(B)$
$$= C_0^3 \times (0{,}8)^3 \times 0{,}4 + C_0^3 \times (0{,}9)^3 \times 0{,}6$$
$$= 0{,}6422$$

c) $$P(A|C_1) = \frac{P(A \cap C_1)}{P(C_1)} = \frac{P(C_1|A) \times P(A)}{P(C_1)} = \frac{C_1^3 \times 0{,}2 \times (0{,}8)^2 \times 0{,}4}{C_1^3 \times 0{,}2 \times (0{,}8)^2 \times 0{,}4 + C_1^3 \times 0{,}1 \times 0{,}9^2 \times 0{,}6} =$$
$$= 0{,}513$$

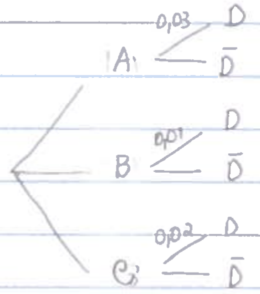
(16)

A – "parafuso escolhido ser M1" — $P(A) = 0{,}30\,P(B)$

B – "parafuso escolhido ser M2" — $P(D|A) = 0{,}03$

D – "ser defeituoso" — $P(D|B) = 0{,}01$

C – "parafuso escolhido ser M3" — $P(D|C) = 0{,}02$



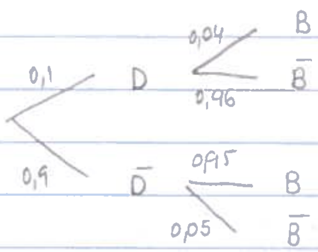
$$P(D) = 0{,}0165 = P(D \cap A) + P(D \cap B) + P(D \cap C)$$
$$= P(D|A).P(A) + P(D|B).P(B) + P(D|C).P(C)$$
$$= 0{,}03 \times (0{,}30\,P(B)) + 0{,}01\,P(B) + 0{,}02\,(1 - P(A) - P(B))$$
$$= 0{,}009\,P(B) + 0{,}01\,P(B) + 0{,}02\,(1 - 0{,}30\,P(B) - P(B))$$
$$= 0{,}019\,P(B) + 0{,}02\,(1 - 1{,}30\,P(B))$$
$$= 0{,}019\,P(B) + 0{,}02 - 0{,}026\,P(B)$$
$$-0{,}0035 = -0{,}007\,P(B)$$
$$P(B) = 0{,}5 \longrightarrow \text{M2 produz } 0{,}5 \times 10.000 = 5\,000$$

$$P(A) = 0{,}30 \times 0{,}50 = 0{,}15 \longrightarrow \text{M1 produz } 0{,}15 \times 10.000 = 1\,500$$
$$P(C) = 1 - 0{,}15 - 0{,}50 = 0{,}35 \longrightarrow \text{M3 produz } 0{,}35 \times 10.000 = 3500$$

(17)

a) D – "ser defeituoso"

B – "artigo bom"



$$P(\bar{B}) = P(\bar{B}|D).P(D) + P(\bar{B}|\bar{D}).P(\bar{D})$$
$$= 0{,}05 \times 0{,}9 + 0{,}96 \times 0{,}1$$
$$= 0{,}141$$

b) $$P(\bar{D}|B) = \frac{P(B \cap \bar{D}) \times P(\bar{D})}{P(B)} = \frac{0{,}95 \times 0{,}9}{0{,}859} = 0{,}995$$

$$P(B) = 1 - P(\bar{B}) = 1 - 0{,}141 = 0{,}859$$

(18)

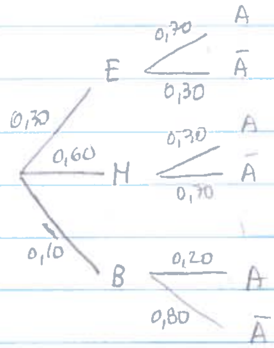
B - "ter Q.I baixo" $P(E) = 0,30$

E - "ter Q.I elevado" $P(M) = 0,60$

M - "ter Q.I médio" $P(M \cap A) = 0,30$

A - "ficar apto" $P(A|B) = 0,20$

$P(A|E) = 0,70$



a) $P(A) = P(A \cap E) + P(A \cap M) + P(A \cap B)$

$= P(A|E).P(E) + 0,30 + P(A|B).P(B)$

$= 0,70 \times 0,30 + 0,30 + 0,20 \times 0,10$

$= 0,53$

b) $P(A|M) = \dfrac{P(A \cap M)}{P(M)} = \dfrac{0,30}{0,60} = 0,5$

c) $(B|\bar{A}) = \dfrac{P(B \cap \bar{A})}{P(\bar{A})} = \dfrac{P(\bar{A}|B).P(B)}{P(\bar{A})} = \dfrac{0,80 \times 0,10}{1 - P(A)} = 0,17$

(19)

A	B
20B 10P	5B 25P

a)

E - "ser executado"

A - "escolher A"

B_i - "bola i ser branca"

$P(\bar{E}) = P(B_1 \cap B_2) + P(B_1 \cap \bar{B}_2 \cap B_3) + P(\bar{B}_1 \cap B_2 \cap B_3)$

$= P(B_1 \cap B_2 \cap A) + P(B_1 \cap \bar{B}_2 \cap B_3 \cap A) + P(\bar{B}_1 \cap B_2 \cap B_3 \cap A) + P(B_1 \cap B_2 \cap \bar{A}) +$

$+ P(B_1 \cap \bar{B}_2 \cap B_3 \cap \bar{A}) + P(\bar{B}_1 \cap B_2 \cap B_3 \cap \bar{A})$

$= P(B_2 | B_1 \cap A) \times P(B_1|A) \times P(A) + P(B_3 | B_1 \cap \bar{B}_2 \cap A) \times P(\bar{B}_2 | B_1 \cap A) \times$

$\times P(B_1|A) \times P(A) + P(B_3 | \bar{B}_1 \cap B_2 \cap A) \times P(B_2 | \bar{B}_1 \cap A) \times P(\bar{B}_1|A) \times P(A) +$

$+ P(B_2 | B_1 \cap \bar{A}) \times P(B_1|\bar{A}) \times P(\bar{A}) + P(B_3 | B_1 \cap \bar{B}_2 \cap \bar{A}) \times P(\bar{B}_2 | B_1 \cap \bar{A}) \times P(B_1|\bar{A}) \times$

$\times P(\bar{A}) + P(B_3 | \bar{B}_1 \cap B_2 \cap \bar{A}) \times P(B_2 | \bar{B}_1 \cap \bar{A}) \times P(\bar{B}_1|\bar{A}) \times P(\bar{A}) =$

$= \dfrac{19}{29} \times \dfrac{20}{30} \times \dfrac{1}{2} + \dfrac{19}{28} \times \dfrac{10}{29} \times \dfrac{20}{30} \times \dfrac{1}{2} + \dfrac{19}{28} \times \dfrac{20}{29} \times \dfrac{10}{30} \times \dfrac{1}{2} +$

$+ \dfrac{4}{29} \times \dfrac{5}{30} \times \dfrac{1}{2} + \dfrac{4}{28} \times \dfrac{25}{29} \times \dfrac{5}{30} \times \dfrac{1}{2} + \dfrac{4}{28} \times \dfrac{5}{29} \times \dfrac{25}{30} \times \dfrac{1}{2} =$

$= 0,4064$

b) $P(A|\bar{E}) = \frac{P(A\cap\bar{E})}{P(\bar{E})} = \frac{P(\bar{B}_1\cap B_2\cap A)+P(B_1\cap\bar{B}_2\cap B_3\cap A)+P(\bar{B}_1\cap B_2\cap B_3\cap A)}{0,4064}$

$= \frac{0,3744}{0,4064} = 0,9212$

20) S_i – "jogador i sobreviver", $i=1,\dots,6$

1 V / 5 F → 2 V / 4 F → 3 V / 3 F

a) $P(S_1) = \frac{5}{6} = 0,8(3)$

$P(S_2) = P(S_2\cap S_1) + P(S_2\cap\bar{S}_1) = P(S_2|S_1)\times P(S_1) + P(S_2|\bar{S}_1)\times P(\bar{S}_1) =$

$P(S_3\cap S_2\cap\bar{S}_1)$ $= \frac{4}{6}\times\frac{5}{6} + 1\times\frac{1}{6} = \frac{13}{18} = 0,7(2)$

$P(S_3) = P(S_3\cap S_2\cap S_1) + P(S_3\cap\bar{S}_2\cap S_1) = P(S_3|S_2\cap S_1)\times P(S_2|S_1)\times P(S_1) +$

$+ P(S_3|\bar{S}_2\cap S_1)\times P(\bar{S}_2|S_1)\times P(S_1) + P(S_3|S_2\cap\bar{S}_1)\times P(S_2|\bar{S}_1)\times P(\bar{S}_1) =$

b) $= \frac{1}{2}\times\frac{4}{6}\times\frac{5}{6} + 1\times\frac{2}{6}\times\frac{5}{6} + 1\times 1\times\frac{1}{6} = 0,7(2)$

b1) $P(\bar{S}_1|S_3) = \frac{P(\bar{S}_1\cap S_3)}{P(S_3)} = \frac{P(S_3|\bar{S}_1)\times P(\bar{S}_1)}{P(S_3)} = \frac{1\times\frac{1}{6}}{0,7(2)} = 0,23$

b2) $P(\bar{S}_2|S_3) = \frac{P(\bar{S}_2\cap S_3)}{P(S_3)} = \frac{P(S_1\cap\bar{S}_2\cap S_3)}{P(S_3)} = \frac{P(S_3|\bar{S}_2\cap S_1)\times P(\bar{S}_2|S_1)\times P(S_1)}{P(S_3)} =$

$= \frac{1\times 2/6\times 5/6}{0,7(2)} = 0,38$

21) S – "igual ou superior a 40 anos"

F – "sexo feminino"

A – "votar partido A"

$P(A|S\cap F) = 0,30$

$P(A|\overline{S\cap F}) = 0,02$

$P(\bar{F}) = 0,45$

$P(\bar{S}) = 0,5$

$P(\bar{F}|\bar{S}) = 0,40$

a)

$P(A) = P(A\cap S\cap F) + P(A\cap\overline{S\cap F}) = P(A|S\cap F)\times P(S\cap F) + P(A|\bar{S}\cup\bar{F})\times P(\bar{S}\cap\bar{F}) =$

$= 0,30\times(1-0,75) + 0,02\times 0,75 = 0,09$

$P(\bar{S}\cup\bar{F}) = P(\bar{S}) + P(\bar{F}) - P(\bar{S}\cap\bar{F})$

$= 0,5 + 0,45 - 0,20$

$= 0,75$

$P(\bar{F}|\bar{S}) = \frac{P(\bar{F}\cap\bar{S})}{P(\bar{S})}$

$P(\bar{F}\cap\bar{S}) = 0,4\times 0,5 = 0,20$

b) $P(\overline{F} \cup \overline{S} \mid A) = \dfrac{P((\overline{F} \cup \overline{S}) \cap A)}{P(A)} = \dfrac{P(\overline{F \cap S} \cap A)}{P(A)} = \dfrac{P(A \mid \overline{F \cap S}) \cdot P(\overline{F \cap S})}{P(A)} =$

$= \dfrac{0{,}02 \times 0{,}75}{0{,}09} = 0{,}167$

22

C – "crise económica"

F – "recorrer a financiamento bancário"

X – "expansão económica"

A – "autofinanciamento"

M – "mercado de capitais"

E – "estabilidade económica"

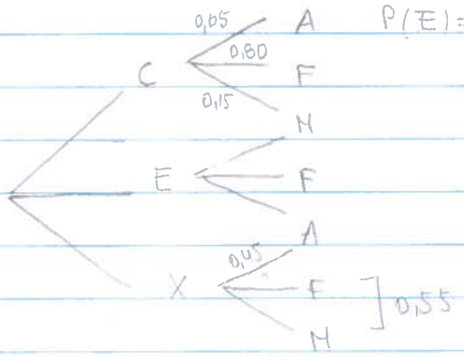
$P(F \mid C) = 0{,}8$ $\quad P(\overline{M} \mid E) = 0{,}75$

$P(A \mid C) = 0{,}05$ $\quad P(E \mid A) = 0{,}58(3)$

$P(C \cap E) = 0{,}20$ $\quad P(F \cup M \mid X) = 0{,}55$

$P(\overline{A} \cap \overline{M} \mid X) = 0{,}20$

$P(E) = 0{,}5$



a) $P(A) = P(A \cap C) + P(A \cap X) + P(A \cap E)$

$= P(C)\,P(A \mid C) + P(X)\,P(A \mid X) + P(E \mid A)\,P(A)$

$= 0{,}25 \times 0{,}05 + 0{,}25 \times 0{,}45 + 0{,}58(3) \times P(A)$

$\simeq 0{,}30$

$P(F \mid C) = \dfrac{P(F \cap C)}{P(C)}$

$P(C) = \dfrac{0{,}20}{0{,}80} = 0{,}25$

$P(X) + P(C) + P(E) = 1 \Leftrightarrow 0{,}5 + 0{,}25 + P(X) = 1$

$P(X) = 0{,}25$

$P(F \cup M \mid X) = \dfrac{P(F \cap X) + P(M \cap X)}{P(X)}$

$P(A \mid X) = 1 - P(F \cup M \mid X)$

$= 1 - 0{,}55$

$= 0{,}45$

$P(F \cap X) + P(M \cap X) = 0{,}1375$

A = F

B = A

C = M

D = C

E = X

F = E

b) $P(\overline{M} \mid E) = 0{,}75$

$1 - P(\overline{M} \mid E) = P(M \mid E) = 0{,}25 = \dfrac{P(M \cap E)}{P(E)} \Leftrightarrow P(M \cap E) = 0{,}125$

$P(E) = P(M \cap E) + P(F \cap E) + P(A \cap E)$

$0{,}5 = 0{,}125 + P(F \cap E) + 0{,}175$

$P(F \cap E) = 0{,}20$

$P(F \mid E) = 0{,}2$

$P(F \mid E) \times P(E) = P(F \cap E)$

$\Leftrightarrow P(F \cap E) = 0{,}05$

$P(E) = P(F \cap C) + P(F \cap E) + P(F \cap X)$

$P(F) = 0{,}45$

$P(F) + \overset{0{,}3}{P(A)} + P(M) = 1$

$P(M) = 0{,}25$

c) $P(C \cup E \mid M) = \dfrac{P((C \cup E) \cap M)}{P(M)} = \dfrac{P(C \cap M) + P(E \cap M)}{P(M)} \Leftrightarrow$

$P(\bar{X} \mid M) = \dfrac{P(\bar{X} \cap M)}{P(M)}$

$P(M) = P(C \cap M) + P(E \cap M) + P(X \cap M)$

$\Leftrightarrow P(C \cap M) + P(E \cap M) = P(M) - P(X \cap M)$

$\Leftrightarrow \dfrac{P(C \cap M) + P(E \cap M)}{P(M)} = \dfrac{P(M) - P(X \cap M)}{P(M)}$

$\Leftrightarrow \dfrac{0{,}25 - 0{,}125}{0{,}25} = 0{,}5 \longrightarrow 50\%$

23

10 bolas

a)

3V
7B

$P(V_2) = P(V_2 \cap V_1) + P(V_2 \cap \bar{V}_1)$

$= P(V_1)\, P(V_2 \mid V_1) + P(\bar{V}_1)\, P(V_2 \mid \bar{V}_1)$

$= \dfrac{3}{10} \times \dfrac{2}{10} + \dfrac{7}{10} \times \dfrac{4}{10}$

$= \dfrac{17}{50}$

b) $P(\bar{V}_1 \cap \bar{V}_2 \mid I)$

24 A - "serem todos rejeitados"

a) $P(A) = (0{,}05)^3 = 0{,}0125$

b) $P(\bar{A}) = 1 - P(A) = 1 - 0{,}0125 = 0{,}9875$

25

a) A = "pelo menos 2 das estradas não estarem bloqueadas"

$$P(A) = 1 - \left(C_1^4 \times 0{,}4 \times 0{,}6^3 + C_0^4 \times (0{,}4)^4 \right)$$

$$= 1 - \left(\frac{4!}{1!\,3!} \times 0{,}4^3 \times 0{,}6^1 + \frac{4!}{0!\,4!} \times (0{,}4)^4 \right)$$

$$= 0{,}8208$$

b) B - "nenhuma das estradas A → B está bloqueada"

X - "A → B bloqueada"

Y - "B → C bloqueada"

Z - "A → C bloqueada"

$$P(\bar{Z}) = C_4^4 \times 0{,}6^4 \times 0{,}4^0 + C_3^4 \times 0{,}6^3 \times 0{,}4^1 + C_1^2 \times 0{,}6^2 \times 0{,}4^2 \times C_1^2$$

$$= 0{,}6^4 + 4 \times 0{,}6^3 \times 0{,}4 + 2 \times 0{,}6^2 \times 0{,}4^2 \times 2$$

$$= 0{,}7056$$

c) $$P(\bar{X} | Z) = \frac{P(\bar{X} \cap Z)}{P(Z)} = \frac{C_2^2 \times 0{,}6^2 \times C_0^2 \times 0{,}4^2 + C_1^2 \times 0{,}6 \times 0{,}4 \times C_0^2 \times 0{,}4^2}{1 - P(\bar{Z})} =$$

$$= 0{,}4565$$

d) $$P(B | \bar{Z}) = \frac{P(B \cap \bar{Z})}{P(\bar{Z})} = \frac{C_0^4 \times 0{,}6^4 + C_1^2 \times 0{,}4 \times 0{,}6 \times C_0^2 \times 0{,}6^2}{0{,}7056} = 0{,}4286$$